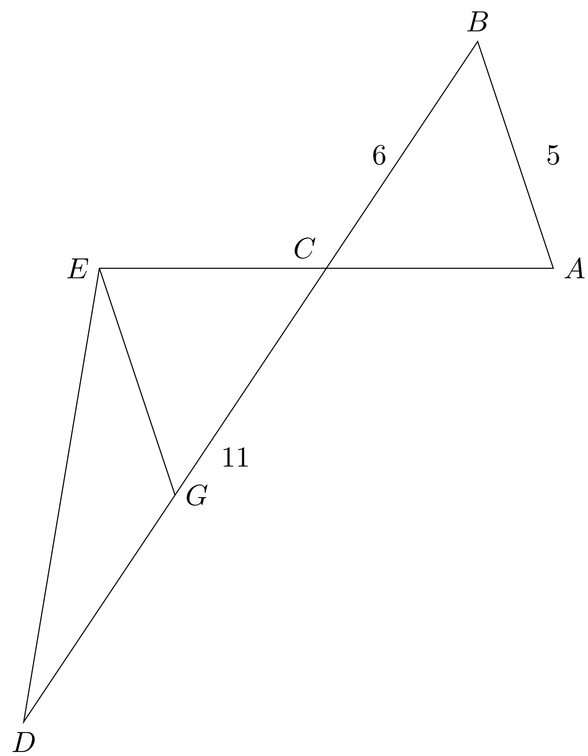
1. Line segment $\overline{AE}$ of length 17 bisects $\overline{DB}$ iat a point C. If $\overline{AB} = 5$, $\overline{BC} = 6$ and $\angle BAC = 78$ degrees, calculate $\angle CDE$.

   Answer: 39

   Solution:

   We draw a diagram as follows:



   We construct line $\overline{EG}$ such that $\angle EGC = \angle ABC = 78$.

   We see that $\overline{EG} = \overline{GD} = 5$ so triangle ΔEGD is isosceles. Therefore, $2\angle GDE = \angle EGC$ and so $\angle CDE = \angle GDE = \boxed{39}$ and we are done.

2. Points A, B, C are chosen on the boundary of a circle with center O so that $\angle BAC$ encloses an arc of 120 degrees. Let D be chosen on $\overline{BA}$ so that $\angle AOD$ is a right angle. Extend $\overline{CD}$ so that it intersects with O again at point P. What is the measure of the arc, in degrees, that is enclosed by $\angle ACP$? Please use the $\tan^{-1}$ function to express your answer.

   Answer: $\mathbf{2(\tan^{-1}(\frac{5\sqrt{3}}{3}) - 30)}$

   Solution: We can draw line $\overline{DO}$ past O, and draw the line from point C parallel to $\overline{AO}$, until they meet at point E. We can deduce that $\angle AOC = 120$ degrees, and so $\angle OCA = 30$, and $\triangle OEC$ is a 30-60-90 triangle, meaning that $\overline{OE} = \frac{\sqrt{3}}{2}$, and $\overline{EC} = \frac{1}{2}$. The measure of the arc we want will be $2 \times \angle ACP$, but we can get the measure of $\angle ECP$ with the $\tan^{-1}$ function, by taking the arctan of $\overline{PE}$ over $\overline{EC}$, and get the measure of $\angle ACP$ by subtracting 30 degrees off, to account for the extra angle given by $\angle ACE$. This gives us our answer of $2(\tan^{-1}(\frac{5\sqrt{3}}{3}) - 30)$

3. Consider a regular polygon with 2^n sides, for $n \geq 2$, inscribed in a circle of radius 1. Denote the area of this polygon by A_n. Compute

$$\prod_{i=2}^{\infty} \frac{A_i}{A_{i+1}}$$

Answer: $\boldsymbol{\frac{2}{\pi}}$ **Solution:** The limit as n goes to infinity of A_n is πr^2 for polygons inscribed in a circle of radius r. Moreover $A_2 = 2r^2$. So

$$\lim_{j \to \infty} \frac{A_2}{A_j} = \frac{2}{\pi}$$

and since all the other terms telescope, we conclude that the infinite product tends to π.