Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Bob has 3 different fountain pens and 11 different ink colors. How many ways can he fill his fountain pens with ink if he can only put one ink in each pen?

2. At the Berkeley Math Tournament, teams are composed of 6 students, each of whom pick two distinct subject tests out of 5 choices. How many different distributions across subjects are possible for a team?

3. Consider the 9×9 grid of lattice points $\{(x, y) \mid 0 \leq x, y \leq 8\}$. How many rectangles with nonzero area and sides parallel to the x, y axes are there such that each corner is one of the lattice points and the point $(4, 4)$ is not contained within the interior of the rectangle? ((4,4) is allowed to lie on the boundary of the rectangle).

4. Alice starts with an empty string and randomly appends one of the digits $2, 0, 1$, or 8 until the string ends with 2018. What is the probability Alice appends less than 9 digits before stopping?

5. Alice and Bob play a game where they start from a complete graph with n vertices and take turns removing a single edge from the graph, with Alice taking the first turn. The first player to disconnect the graph loses. Compute the sum of all n between 2 and 100 inclusive such that Alice has a winning strategy. (A complete graph is one where there is an edge between every pair of vertices.)

6. Compute
$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \binom{i+j}{i} 3^{-(i+j)}.$$

7. Let S be the set of line segments between any two vertices of a regular 21-gon. If we select two distinct line segments from S at random, what is the probability they intersect? Note that line segments are considered to intersect if they share a common vertex.

8. Moor and nine friends are seated around a circular table. Moor starts out holding a bottle, and whoever holds the bottle passes it to the person on his left or right with equal probability until everyone has held the bottle. Compute the expected distance between Moor and the last person to receive the bottle, where distance is the fewest number of times the bottle needs to be passed in order to go back to Moor.

9. Let S be the set of integers from 1 to 13 inclusive. A permutation of S is a function $f : S \to S$ such that $f(x) \neq f(y)$ if $x \neq y$. For how many distinct permutations f does there exists an n such that $f^n(i) = 13 - i + 1$ for all i.

10. Consider a $2 \times n$ grid where each cell is either black or white, which we attempt to tile with 2×1 black or white tiles such that tiles have to match the colors of the cells they cover. We first randomly select a random positive integer N where N takes the value n with probability 2^{-n}. We then take a $2 \times N$ grid and randomly color each cell black or white independently with equal probability. Compute the probability the resulting grid has a valid tiling.