Time limit: 60 minutes.
Instructions: For this test, you work in teams of five to solve 20 short answer questions.
No calculators.

1. If $x^2 = 7$, what is $x^4 + x^2 + 1$?

2. Richard and Alex are competing in a 150-meter race. If Richard runs at a constant speed of 5 meters per second and Alex runs at a constant speed of 3 meters per second, how many more seconds does it take for Alex to finish the race?

3. David and Emma are playing a game with a chest of 100 gold coins. They alternate turns, taking one gold coin if the chest has an odd number of gold coins or taking exactly half of the gold coins if the chest has an even number of gold coins. The game ends when there are no more gold coins in the chest. If Emma goes first, how many gold coins does Emma have at the end?

4. What is the only 3-digit perfect square whose digits are all different and whose units digit is 5?

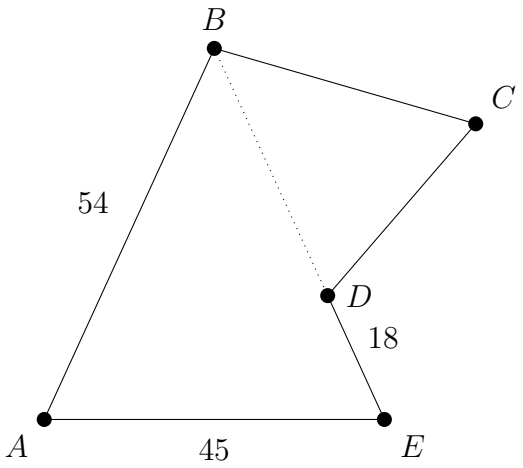
5. In regular pentagon $ABCDE$, let F be the midpoint of $\overline{AB}$, G be the midpoint of $\overline{CD}$, and H be the midpoint of $\overline{AE}$. What is the measure of $\angle FGH$ in degrees?

6. Water enters at the left end of a pipe at a rate of 1 liter per 35 seconds. Some of the water exits the pipe through a leak in the middle. The rest of the water exits from the right end of the pipe at a rate of 1 liter per 36 seconds. How many **minutes** does it take for the pipe to leak a liter of water?

7. Carson wants to create a wire frame model of a right rectangular prism with a volume of 2022 cubic centimeters, where strands of wire form the edges of the prism. He wants to use as much wire as possible. If Carson also wants the length, width, and height in centimeters to be **distinct** whole numbers, how many centimeters of wire does he need to create the prism?

8. How many ways are there to fill the unit squares of a 3×5 grid with the digits 1, 2, and 3 such that every pair of squares that share a side differ by exactly 1?

9. In pentagon $ABCDE$, $AB = 54$, $AE = 45$, $DE = 18$, $\angle A = \angle C = \angle E$, D is on line segment $\overline{BE}$, and line $\overleftrightarrow{BD}$ bisects angle $\angle ABC$, as shown in the diagram below. What is the perimeter of pentagon $ABCDE$?



10. If x and y are nonzero real numbers such that $\frac{7}{x} + \frac{8}{y} = 91$ and $\frac{6}{x} + \frac{10}{y} = 89$, what is the value of $x + y$?

11. Hilda and Marianne play a game with a shuffled deck of 10 cards, numbered from 1 to 10. Hilda draws five cards, and Marianne picks up the five remaining cards. Hilda observes that she does not have any pair of consecutive cards - that is, no two cards have numbers that differ by exactly 1. Additionally, the sum of the numbers on Hilda's cards is 1 less than the sum of the numbers on Marianne's cards. Marianne has exactly one pair of consecutive cards - what is the sum of this pair?

12. Regular hexagon $AUSTIN$ has side length 2. Let M be the midpoint of line segment $\overline{ST}$. What is the area of pentagon $MINUS$?

13. At a collector's store, plushes are either small or large and cost a positive integer number of dollars. All small plushes cost the same price, and all large plushes cost the same price. Two small plushes cost exactly one dollar less than a large plush. During a shopping trip, Isaac buys some plushes from the store for 59 dollars. What is the smallest number of dollars that the small plush could not possibly cost?

14. Four fair six-sided dice are rolled. What is the probability that the median of the four outcomes is 5?

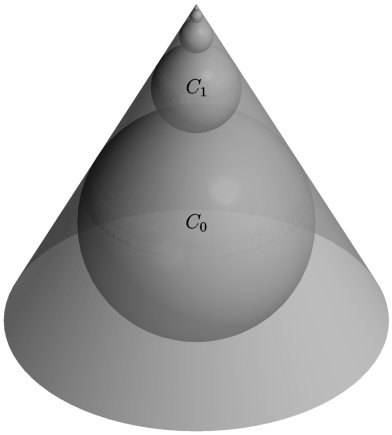
15. Suppose $x_1, x_2, \ldots, x_{2022}$ is a sequence of real numbers such that:

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_2 + x_3 &= 2 \\ &\vdots \\ x_{2021} + x_{2022} &= 2021. \end{aligned}$$

    If $x_1 + x_{499} + x_{999} + x_{1501} = 222$, then what is the value of x_{2022}?

16. A cone has radius 3 and height 4. An infinite number of spheres are placed in the cone in the following way: sphere C_0 is placed inside the cone such that it is tangent to the base of the cone and to the curved surface of the cone at more than one point, and for $i \geq 1$, sphere C_i is placed such that it is externally tangent to sphere C_{i-1} and internally tangent to more than one point of the curved surface of the cone. If V_i is the volume of sphere C_i, compute

$$V_0 + V_1 + V_2 + \cdots .$$



17. Call an ordered pair, (x, y), *relatable* if x and y are positive integers where y divides 3600, x divides y, and $\frac{y}{x}$ is a prime number. For every relatable ordered pair, Leanne wrote down the positive difference of the two terms of the pair. What is the sum of the numbers she wrote down?

18. Let r, s, and t be the three roots of $P(x) = x^3 - 9x - 9$. Compute the value of

$$(r^3 + r^2 - 10r - 8)(s^3 + s^2 - 10s - 8)(t^3 + t^2 - 10t - 8).$$

19. Compute the number of ways to color the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$, and 9 red, blue, or green such that:

    (a) every prime integer has at least one digit that is not blue, and

    (b) every composite integer has at least one digit that is not green.

    Note that 0 is not composite. For example, since 12 is composite, either the digit 1, the digit 2, or both must be not green.

20. Pentagon $ABCDE$ has $AB = DE = 4$ and $BC = CD = 9$ with $\angle ABC = \angle CDE = 90^\circ$, and there exists a circle tangent to all five sides of the pentagon. What is the length of segment $\overline{AE}$?