1. If the pairwise sums of the three numbers x, y, and z are 22, 26, and 28, what is $x + y + z$?

   Answer: 38

   Solution: Let $x+y = 22, x+z = 26$, and $y+z = 28$. Adding up the equations gives $2x+2y+2z = 76 \implies x + y + z = \boxed{38}$.

2. Suhas draws a quadrilateral with side lengths 7, 15, 20, and 24 in some order such that the quadrilateral has two opposite right angles. Find the area of the quadrilateral.

   Answer: 234

   Solution: Observing that 7-24-25 and 15-20-25 are pythagorean triples, we find that Suhas's quadrilateral $ABCD$ has $AB = 7, BC = 24, CD = 15$, and $DA = 20$. Consequently, $\angle ABC = \angle CDA = 90$ degrees. Using the pythagorean theorem, we find that no other configuration of Suhas's quadrilateral is possible. In particular, we find that the two right triangles can neither have legs 7 and 15, and 20 and 24, respectively, nor have legs 7 and 20, and 15 and 24, respectively. This is because $7^2 + 15^2 < 20^2 + 24^2$, and $7^2 + 20^2 < 15^2 + 24^2$ so in particular $7^2 + 15^2 \neq 20^2 + 24^2$ and $7^2 + 20^2 \neq 15^2 + 24^2$. The area of this quadrilateral is the sum of the areas of triangles ΔABC and ΔCDA, which is $\frac{1}{2} \cdot 7 \cdot 24 + \frac{1}{2} \cdot 15 \cdot 20 = \boxed{234}$.

3. Let $(n)^*$ denote the sum of the digits of n. Find the value of $((((985^{998})^*)^*)^*)^*$.

   Answer: 7

   Solution: Let $N = 985^{998}$. Notice that the remainder of the sum of the digits of n when divided by 9 is equal to the remainder when n is divided by 9. Since $N = 985^{998} < 1000^{1000}$, N has at most 3000 digits, so the sum of digits is at most $9 \cdot 3000 = 27000$ and consequently N^* has at most 5 digits. Because $(N)^*$ has at most 5 digits, the sum of the digits of $(N)^*$ is at most $9 \cdot 5 = 45$. By a similar argument, we find that $((((N)^*)^*)^*$ is at most 18, and $((((N)^*)^*)^*)^*$ is at most 9, so $((((N)^*)^*)^*)^*$ is just equal to its remainder when divided by 9. We thus have $985^{998} \equiv (4)^{998} \equiv 4^2 \equiv \boxed{7} \pmod 9$ by Euler's theorem.

4. Everyone wants to know Andy's locker combination because there is a golden ticket inside. His locker combination consists of 4 non-zero digits that sum to an even number. Find the number of possible locker combinations that Andy's locker can have.

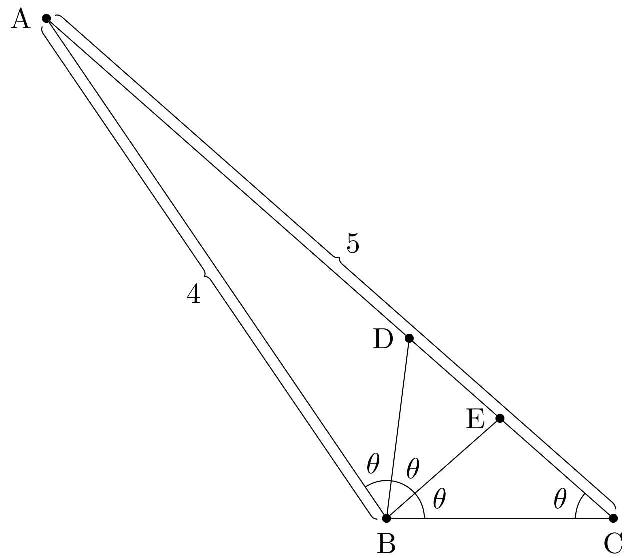
   Answer: 3281

   Solution: Andy's locker combination can be four odd digits, four even digits, or two odd digits and two even digits. The total number of locker combinations with four even digits is $5 \cdot 5 \cdot 5 \cdot 5 = 625$ possibilities. The total number of locker combinations with four odd digits is $4 \cdot 4 \cdot 4 \cdot 4 = 256$ digits. The total number of locker combinations with two odd digits and two odd digits is $\binom{4}{2} \cdot 5 \cdot 5 \cdot 4 \cdot 4 = 2400$ because there are $\binom{4}{2}$ ways to choose the two even digits and $5 \cdot 5 \cdot 4 \cdot 4$ ways to pick the digits themselves. Adding up all these cases yields $625 + 256 + 2400 = \boxed{3281}$ possible locker combinations.

5. In triangle ABC, $\angle ABC = 3\angle ACB$. If $AB = 4$ and $AC = 5$, compute the length of BC.

   Answer: $\mathbf{\frac{3}{2}}$ **OR** $\mathbf{1\frac{1}{2}}$ **OR 1.5**

   Solution:



Construct D and E lie on line segment AC such that $\angle ABD = \angle DBE = \angle EBC$. Let $\angle ACB = \theta$. Then we have $\angle ABC = 3\theta$ and $\angle ABD = \angle DBE = \angle EBC = \theta$. First, note that $\angle AEB = \angle EBC + \angle ECB = 2\theta = \angle ABE$, so $AB = AE = 4$ and $BE = EC = 1$.

Notice that $\angle DEB = 2\theta = \angle DBC$ and $\angle DBE = \theta = \angle DCB$. Thus, by AA similarity, triangle DBE similar to DCB, so $\frac{DB}{DE} = \frac{DC}{DB} \implies DB^2 = DE \cdot DC$.

By the Angle Bisector Theorem, $\frac{AD}{DE} = 4$. Since $AE = AD + DE = 4$, so $AD = \frac{16}{5}$ and $DE = \frac{4}{5}$.

Thus, $BD = \sqrt{DE \cdot DC} = \sqrt{\frac{4}{5}\left(\frac{4}{5} + 1\right)} = \sqrt{\frac{36}{25}} = \frac{6}{5}$. Finally, by similar triangles, $\frac{AB}{BD} = \frac{AC}{BC}$, so $BC = \frac{BD \cdot AC}{AB} = \frac{\frac{6}{5} \cdot 5}{4} = \boxed{\frac{3}{2}}$.