1. If Clark wants to divide 100 pizzas among 25 people so that each person receives the same number of pizzas, how many pizzas should each person receive?

   Answer: 4

   Solution: Since there are 25 people and 100 pizzas, each person should receive $\frac{100}{25} = \boxed{4}$ pizzas.

2. In a group of 3 people, every pair of people shakes hands once. How many handshakes occur?

   Answer: 3

   Solution: There are $\binom{3}{2} = 3$ pairs of people, so there are $\boxed{3}$ handshakes in total.

3. Dylan and Joey have 14 costumes in total. Dylan gives Joey 4 costumes, and Joey now has the number of costumes that Dylan had before giving Joey any costumes. How many costumes does Dylan have now?

   Answer: 5

   Solution: Let d be the number of costumes that Dylan ends up with. Then Joey ends up with $14 - d$ costumes, and Dylan starts off with $14 - d$ costumes. Dylan gives Joey 4 costumes, so we have that $(14 - d) - d = 4$, and $d = \boxed{5}$.

4. At Banjo Borger, a burger costs 7 dollars, a soda costs 2 dollars, and a cookie costs 3 dollars. Alex, Connor, and Tony each spent 11 dollars on their order, but none of them got the same order. If Connor bought the most cookies, how many cookies did Connor buy?

   Answer: 3

   Solution: There are only three ways to write 11 as a sum of 7s, 2s, and 3s: $11 = 7 + 2 + 2 = 2 + 3 + 3 + 3 = 2 + 2 + 2 + 2 + 3$, corresponding to orders of 1 burger and 2 sodas, 1 soda and 3 cookies, and 4 sodas and 1 cookie. Therefore, Connor got $\boxed{3}$ cookies.

5. Joey, James, and Austin stand on a large, flat field. If the distance from Joey to James is 30 and the distance from Austin to James is 18, what is the minimal possible distance from Joey to Austin?

   Answer: 12

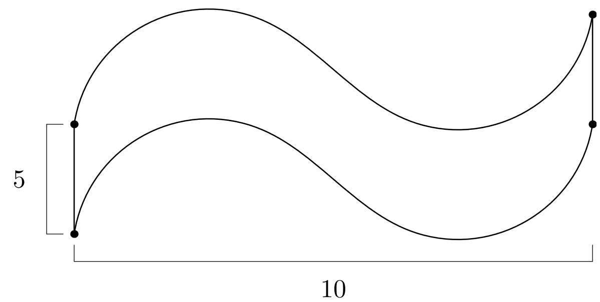
   Solution: By the Triangle Inequality, the minimal distance occurs when Joey, Austin, and James lie on a line with Austin between Joey and James. This configuration gives the minimal distance from Joey to Austin as $30 - 18 = \boxed{12}$.

6. If the first and third terms of a five-term arithmetic sequence are 3 and 8, respectively, what is the sum of all 5 terms in the sequence?

   Answer: 40

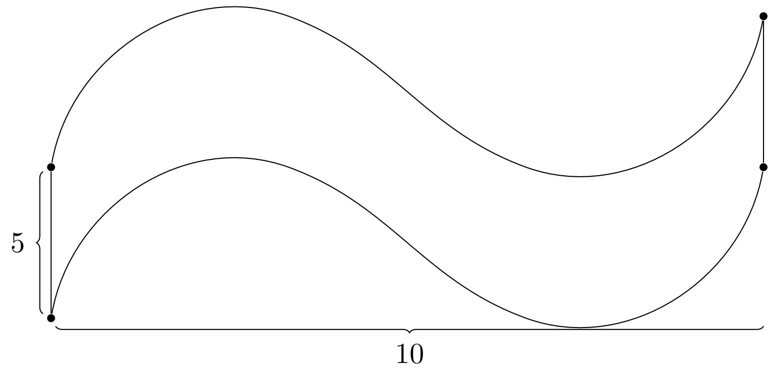
   Solution: There are five terms and the middle term is 8, so the sum is $5 \cdot 8 = \boxed{40}$.

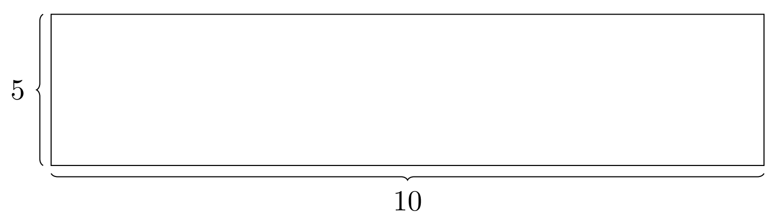
7. What is the area of the S-shaped figure below, which has constant vertical height 5 and width 10?



Answer: 50

Solution:





This is Archimedes' principle! Each of the vertical strips lying in the S-shaped figure is of length 5. When we flatten the figure to a rectangle of height 5 and width 10, the area will remain the same. Thus, the area is $5 \cdot 10 = \boxed{50}$.

8. If the side length of square A is 4, what is the perimeter of square B, formed by connecting the midpoints of the sides of A?

   Answer: $\mathbf{8\sqrt{2}}$

   Solution: The side length of square B is $\sqrt{2^2 + 2^2} = 2\sqrt{2}$. Thus, its perimeter is $4 \cdot 2\sqrt{2} = \boxed{8\sqrt{2}}$.

9. The Chan Shun Auditorium at UC Berkeley has room number 2050. The number of seats in the auditorium is a factor of the room number, and there are between 150 and 431 seats, inclusive. What is the sum of all of the possible numbers of seats in Chan Shun Auditorium?

   Answer: 615

   Solution: The factors of 2050 between 150 and 431 are 205 and 410, which sum to $\boxed{615}$.

10. Krishna has a positive integer x. He notices that x^2 has the same last digit as x. If Krishna knows that x is a prime number less than 50, how many possible values of x are there?

Answer: 4

Solution: We only care about the last digit of x, which could be any digit from 0 to 9, inclusive. Checking all cases gives that x must end in 0, 1, 5, or 6. Prime numbers cannot end with 0 or 6, since they would be divisible by 2. Moreover, any number that ends with 5 is divisible by 5, so the only prime number that ends with 5 is the number 5. The only other possible values for x are primes less than 50 that have last digit 1, of which we find 11 and 31 (1 is not a prime!), and 41, giving $\boxed{4}$ total solutions.

11. Jing Jing the Kangaroo starts on the number 1. If she is at a positive integer n, she can either jump to $2n$ or to the sum of the digits of n. What is the smallest positive integer she cannot reach no matter how she jumps?

Answer: 3

Solution: The sum of the digits modulo 3 is equal to n modulo 3. Thus, taking the sum of the digits of n or doubling the number n cannot change the fact that n isn't a multiple of 3. In addition, Jing Jing can reach 1 and 2 since she starts at 1 and since $2 = 2 \cdot 1$. It thus follows that $\boxed{3}$ is the minimum positive integer Jing Jing cannot reach.

12. Sylvia is 3 units directly east of Druv and runs twice as fast as Druv. When a whistle blows, Druv runs directly north, and Sylvia runs along a straight line. If they meet at a point a distance d units away from Druv's original location, what is the value of d?

Answer: $\sqrt{3}$

Solution: Druv travels a distance d, and Sylvia runs at twice the pace as Druv, so Sylvia travels a distance $2d$. Thus, by the Pythagorean Theorem, $(2d)^2 = d^2 + 3^2$, which gives that $d = \boxed{\sqrt{3}}$.

13. If x is a real number such that $\sqrt{x} + \sqrt{10} = \sqrt{x + 20}$, compute x.

Answer: $\frac{5}{2}$ or 2.5

Solution: Squaring both sides, we get $(x + 10) + 2\sqrt{10x} = x + 20$, so $2\sqrt{10x} = 10$ and $x = \frac{5}{2}$.

14. Compute the number of rearrangements of the letters in LATEX such that the letter T comes before the letter E and the letter E comes before the letter X. For example, TLEAX is a valid rearrangement, but LAETX is not.

Answer: 20

Solution: Of the $3! = 6$ orders that the letters T, E, X can be in, only 1 of them has the letters in the desired order. There are $5! = 120$ ways to arrange the letters L, A, T, E, X and for each ordering of L, A, T, E, X, there are an equal number of arrangements of the letters of LATEX such that the letters of TEX appear in any particular order. Hence the number of rearrangements is $5! \cdot \frac{1}{3!} = \boxed{20}$ strings.

15. How many integers n greater than 2 are there such that the degree measure of each interior angle of a regular n-gon is an even integer?

Answer: 16

Solution: We know the angle measure of a regular n-gon is $180 - \frac{360}{n}$. For this to be even, $\frac{360}{n}$ must be even. Thus n must be a factor of 180 (since $\frac{360}{n} \cdot \frac{1}{2}$ must be an integer). Since

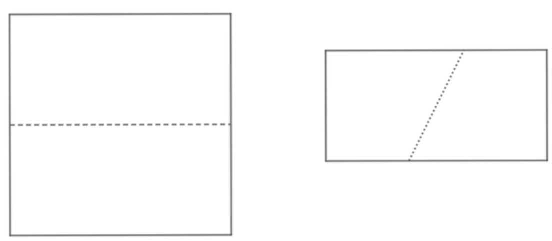
$180 = 2^2 \cdot 3^2 \cdot 5$, there are $3 \cdot 3 \cdot 2 = 18$ factors. Ignoring the factors 1 and 2 since $n > 2$, so there are a total of $\boxed{16}$ such n.

16. Students are being assigned to faculty mentors in the Berkeley Math Department. If there are 7 distinct students and 3 distinct mentors, and each student has exactly one mentor, in how many ways can students be assigned to mentors such that each mentor has at least one student?

Answer: 1806

Solution: There are a total of 3^7 total assignments. To count the number of valid assignments, we subtract out the number of assignments that map to only one or two mentors. Mapping to one mentor, there are 3 assignments (all students are assigned to that mentor). For two mentors, there are $\binom{3}{2} \cdot (2^7 - 2)$: $\binom{3}{2}$ ways to select the two mentors that get students and $2^7 - 2$ ways to perform the mappings. The reason we subtracted two is to subtract out the case where all three students select the same mentor, which we already accounted for. Thus, the final answer is $3^7 - (3 \cdot (2^7 - 2) + 3) = 3^7 - 3 \cdot 2^7 - 3 = \boxed{1806}$.

17. Karthik has a paper square of side length 2. He folds the square along a crease that connects the midpoints of two opposite sides (as shown in the left diagram, where the dotted line indicates the fold). He takes the resulting rectangle and folds it such that one of its vertices lands on the vertex that is diagonally opposite. Find the area of Karthik's final figure.



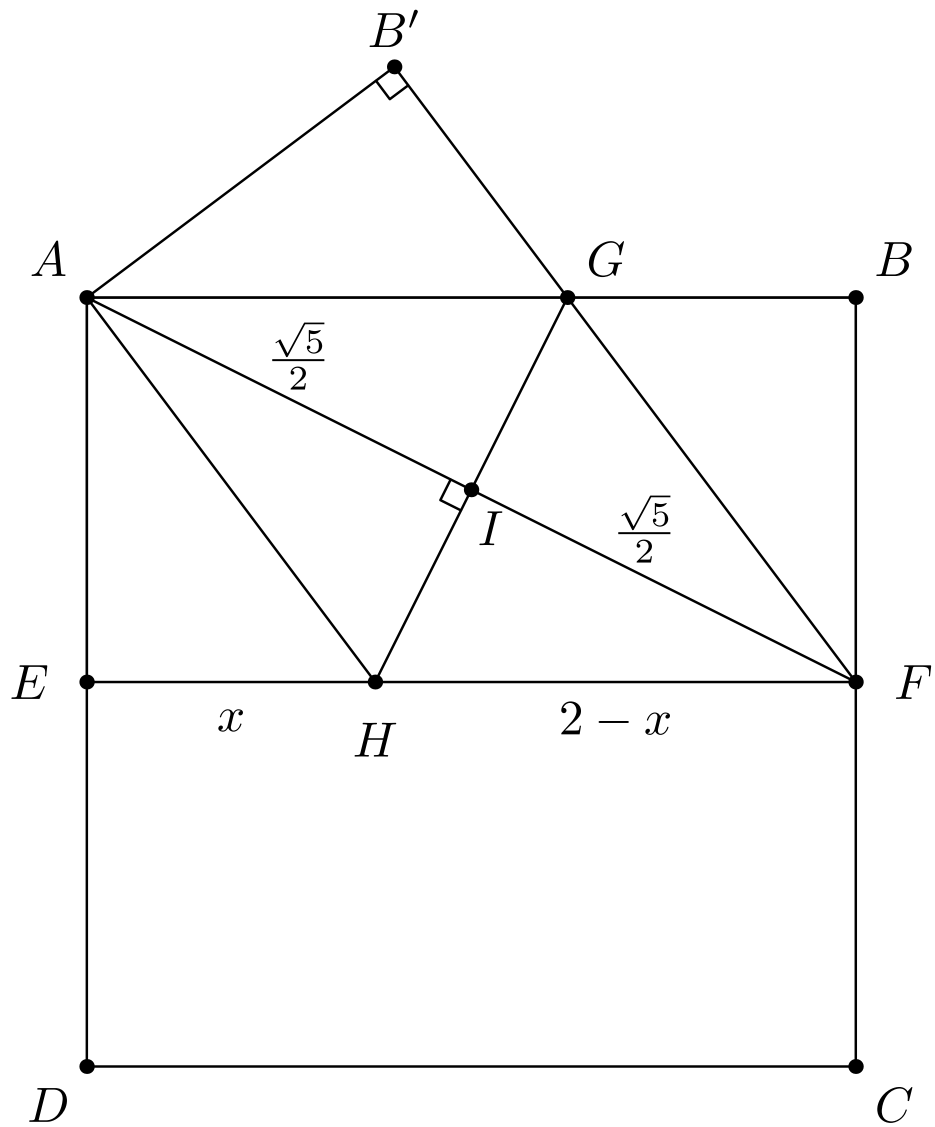
Answer: $\frac{11}{8}$ OR $1\frac{3}{8}$ OR 1.375

Solution: Label the original square $ABCD$, and let E be the midpoint of AD and F be the midpoint of BC. Then, Karthik's figure after the first fold is $ABFE$. The second crease is perpendicular to diagonal AF, and label its intersection with AB as G and with EF as H. Let the intersection of GH and AF be I. We must compute the area of $AB'GHE$ where B' is the image of B after the fold (i.e. B' is the reflection of B about line HG). Note that $\text{area}(AB'GHE) = 2 \cdot \text{area}(AEH) + \text{area}(AGH)$. If $EH = x$, then $\triangle IFH \sim \triangle EFA$ and consequently $\frac{2-x}{\frac{\sqrt{5}}{2}} = \frac{\sqrt{5}}{2}$. Solving for x yields $x = \frac{3}{4} \implies \text{area}(AEH) = \frac{1}{2} \cdot \frac{3}{4} \cdot 1 = \frac{3}{8}$ and $\text{area}(AGH) = \frac{\text{area}(ABFE) - 2 \cdot \text{area}(AEH)}{2} = \frac{2 - \frac{3}{4}}{2} = \frac{5}{8}$. Thus, our final area is $\frac{3}{4} + \frac{5}{8} = \boxed{\frac{11}{8}}$.

18. Sally is inside a pen consisting of points (a, b) such that $0 \leq a, b \leq 4$. If she is currently on the point (x, y), she can move to either $(x, y+1)$, $(x, y-1)$, or $(x+1, y)$. Given that she cannot revisit any point she has visited before, find the number of ways she can reach $(4, 4)$ from $(0, 0)$.

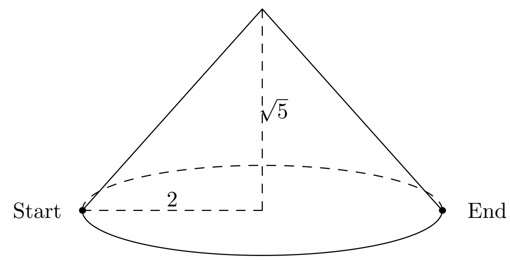
Answer: 625

Solution: Let increasing x and y represent moving right and upwards, respectively. Once Sally moves right she cannot move left again. Now consider Sally's movement within each column (each fixed x). At any given y-coordinate in that column, she can either travel right or travel



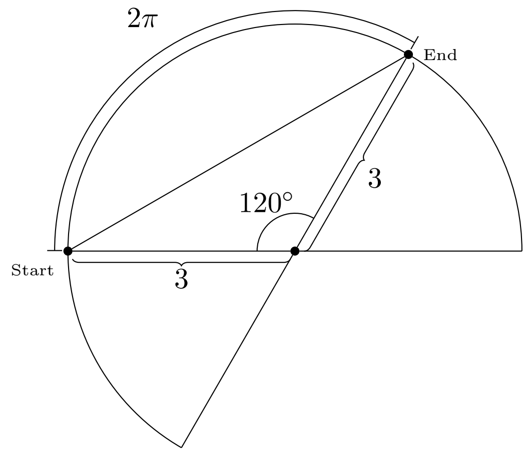
vertically and then travel right. Since her path cannot overlap itself, for any y_1, y_2-coordinates in each column, there is exactly one way to travel from (x, y_1) to (x, y_2). Thus, for each of the first 4 columns Sally has 5 choices of y-coordinate on which she will travel right - once she reaches the column $x = 4$, there is only one way to travel up to $(4, 4)$. Thus, our answer is $5^4 = \boxed{625}$.

19. An ant sits on the circumference of the circular base of a party hat (a cone without a circular base for the ant to walk on) of radius 2 and height $\sqrt{5}$. If the ant wants to reach a point diametrically opposite of its current location on the hat, what is the minimum possible distance the ant needs to travel?



Answer: $3\sqrt{3}$

Solution:



"Unfold" the party hat by making a straight cut through the apex and a point on the circular base. The flattened party hat is a sector of a circle with radius 3. The total length of the arc of the sector is $2 \cdot 2\pi = 4\pi$, and the circumference of the circle is 6π, so the angle that the sector takes should be $360° \cdot \frac{4\pi}{6\pi} = 240°$. Similarly, the ant's desired location is $360° \cdot \frac{2\pi}{6\pi} = 120°$ away from its current location. Since the shortest path between these two points is the straight line between them, we draw an isosceles triangle whose two equal legs are radii (and thus of length 3) and whose vertex angle is 120°. The shortest distance to the desired location is then the length of the triangle's third side. By dropping an altitude to the third side and forming two $30-60-90$ triangles, we see that the length of the third side is $\boxed{3\sqrt{3}}$.

20. If

$$f(x) = \frac{2^{19}x + 2^{20}}{x^2 + 2^{20}x + 2^{20}},$$

find the value of $f(1) + f(2) + f(4) + f(8) + \cdots + f(2^{20})$.

Answer: $\frac{21}{2}$ OR $10\frac{1}{2}$ OR 10.5

Solution: Notice that $f(2^{20}/x) = \dfrac{2^{19} \cdot 2^{20}/x + 2^{20}}{(2^{20})^2/x^2 + (2^{20})^2/x + 2^{20}} = \dfrac{2^{19}x + x^2}{x^2 + 2^{20}x + 2^{20}}$, so $f(x) + f(2^{20}/x) = 1$. Thus the desired sum is $(f(1) + f(2^{20})) + (f(2) + f(2^{19})) + \ldots (f(2^9) + f(2^{11})) + \frac{1}{2}(f(2^{10}) + f(2^{10})) = \boxed{\dfrac{21}{2}}$.