Countdown Round

Berkeley mini Math Tournament

16 November 2013

Problem 1

You roll 3 dice. What is the probability that the product of the outcomes is a factor of 64?

Problem 1

You roll 3 dice. What is the probability that the product of the outcomes is a factor of 64?

The answer is $\frac{1}{8}$.

Problem 2

Problem 2

Suppose that points A, B, C, D lie on a circle in that order. Segments AC and BD intersect at E. Given that $AE = 1$, $AD = 2$, and $BE = 3$, find all possible values for the length of BC.

Problem 2

Suppose that points A, B, C, D lie on a circle in that order. Segments AC and BD intersect at E. Given that $AE = 1$, $AD = 2$, and $BE = 3$, find all possible values for the length of BC.

The answer is 6.

Problem 3

Nine math students take a 100 point test. The average of their scores is 79. If a tenth student takes the test, what is the minimum score that she needs to get to raise the class average score to 81?

Problem 3

Nine math students take a 100 point test. The average of their scores is 79. If a tenth student takes the test, what is the minimum score that she needs to get to raise the class average score to 81?

The answer is 99.

Problem 4

Find the unit's digit of the expression
$2013^1 + 2013^2 + 2013^3 + ... + 2013^{2013}$.

Problem 4

Find the unit's digit of the expression
$2013^1 + 2013^2 + 2013^3 + ... + 2013^{2013}$.

The answer is 3.

Problem 5

Find the integer closest to $\pi^3 + \pi^2 + \pi + 1$.

Problem 5

Find the integer closest to $\pi^3 + \pi^2 + \pi + 1$.

The answer is 45.

Problem 6

Three circles with radii 1, 2, and 3 are mutually externally tangent. Find the area of the triangle formed by their centers.

Problem 6

Problem 6

Three circles with radii 1, 2, and 3 are mutually externally tangent. Find the area of the triangle formed by their centers.

The answer is 6.

Problem 7

If $A = (2, 6)$, $P = (5, 15)$ and $B = (7, 21)$, what is the ratio of the lengths $AP : PB$?

Problem 7

If $A = (2, 6)$, $P = (5, 15)$ and $B = (7, 21)$, what is the ratio of the lengths $AP : PB$?

The answer is $3 : 2$.

Problem 8

How many three digit numbers are there with an odd number of odd digits?

Problem 8

How many three digit numbers are there with an odd number of odd digits?

The answer is 450.

Problem 9

A square and a triangle have equal area. Let P_s be the perimeter of the square, and P_t be the perimeter of the triangle. What is the maximum possible value of $\frac{P_s^2}{P_t^2}$?

Problem 9

A square and a triangle have equal area. Let P_s be the perimeter of the square, and P_t be the perimeter of the triangle. What is the maximum possible value of $\frac{P_s^2}{P_t^2}$?

The answer is $\frac{4\sqrt{3}}{9}$.

Problem 10

m and n are positive integers satisfying $m^2 + n^2 = 2000$. What is the maximum possible value of mn?

Problem 10

m and n are positive integers satisfying $m^2 + n^2 = 2000$. What is the maximum possible value of mn?

The answer is 800.

Problem 11

You are given 9 coins of a special currency, of denominations 1 through 9. In how many ways can you distribute the coins into 5 stacks such that each stack has the same value? Assume the stacks are unordered.

Problem 11

You are given 9 coins of a special currency, of denominations 1 through 9. In how many ways can you distribute the coins into 5 stacks such that each stack has the same value? Assume the stacks are unordered.

The answer is 1.

Problem 12

Every minute it is in your mouth, a stick of gum loses 20% of its current flavor. To the nearest minute, how long will the stick of gum have more than half its original flavor?

Problem 12

Every minute it is in your mouth, a stick of gum loses 20% of its current flavor. To the nearest minute, how long will the stick of gum have more than half its original flavor?

The answer is 3.

Problem 13

Billfred painstakingly wrote all the odd numbers from 1 to 199 (inclusive) on the board. Fredbob accidentally erased the first n odd numbers. If the sum of the remaining numbers on the board is 9900, find n.

Problem 13

Billfred painstakingly wrote all the odd numbers from 1 to 199 (inclusive) on the board. Fredbob accidentally erased the first n odd numbers. If the sum of the remaining numbers on the board is 9900, find n.

The answer is 10.

Problem 14

An infinitely large army of soldiers lies along the line $y = x + 2$ and advances in the positive x-direction at a constant speed of 1 unit per second. If Jacob is at $(0, 0)$ and is running away from the soldiers at a constant speed of 2 units per second, locate where he will be in 2 seconds, assuming he takes the route distancing himself farthest from the nearest soldier.

Problem 14

An infinitely large army of soldiers lies along the line $y = x + 2$ and advances in the positive x-direction at a constant speed of 1 unit per second. If Jacob is at $(0, 0)$ and is running away from the soldiers at a constant speed of 2 units per second, locate where he will be in 2 seconds, assuming he takes the route distancing himself farthest from the nearest soldier.

The answer is $(2\sqrt{2}, -2\sqrt{2})$.

Problem 15

What is the sum of all positive integers less than 105 that are relatively prime to 105?

Problem 15

What is the sum of all positive integers less than 105 that are relatively prime to 105?

The answer is 2520.

Problem 16

Problem 16

In how many ways can you rearrange the letters in *ADJACENT* such that T is adjacent to at least one A?

Problem 16

In how many ways can you rearrange the letters in *ADJACENT* such that T is adjacent to at least one A?

The answer is 9360.

Problem 17

Suppose $f(x) = x^2 + 28x - 2013$.

What is the largest positive integer a such that $f(a)$ is not positive?

Problem 17

Suppose $f(x) = x^2 + 28x - 2013$.

What is the largest positive integer a such that $f(a)$ is not positive?

The answer is 33.

Problem 18

Given triangle ABC with $\overline{AB} = 6$, $\overline{BC} = 8$, and $\angle ABC = 90°$. If BD is the altitude from B to AC, what is the length of AD?

Problem 18

Given triangle ABC with $\overline{AB} = 6$, $\overline{BC} = 8$, and $\angle ABC = 90°$. If BD is the altitude from B to AC, what is the length of AD?

The answer is $\frac{18}{5}$.

Problem 19

Three rectangles with integer side lengths are placed adjacent to each other to create a larger rectangle R. If the sum of the perimeters of the three rectangles is 42, what is the minimal possible area of R?

Problem 19

Three rectangles with integer side lengths are placed adjacent to each other to create a larger rectangle R. If the sum of the perimeters of the three rectangles is 42, what is the minimal possible area of R?

The answer is 18.

Problem 20

Each of the following self-referential logical statements has a truth value. Determine how many of them are true.

1. This statement is both true and false.
2. This statement is either true or false.
3. This statement is neither true nor false.
4. If this statement is true, then this statement is false.

Problem 20

Each of the following self-referential logical statements has a truth value. Determine how many of them are true.

1. This statement is both true and false.
2. This statement is either true or false.
3. This statement is neither true nor false.
4. If this statement is true, then this statement is false.

The answer is 1.

Problem 21

How many non-congruent triangles with integer side lengths have perimeter 8?

Problem 21

How many non-congruent triangles with integer side lengths have perimeter 8?

The answer is 1.

Problem 22

A two-digit decimal number XY_{10} is reversed to YX_{16} in hexadecimal. Find XY_{10}.

Problem 22

A two-digit decimal number XY_{10} is reversed to YX_{16} in hexadecimal. Find XY_{10}.

The answer is 53.

Problem 23

Bob multiplies a number by 2 to get 100 as the answer. However, instead of multiplying he was supposed to add the 2. What is the correct answer?

Problem 23

Bob multiplies a number by 2 to get 100 as the answer. However, instead of multiplying he was supposed to add the 2. What is the correct answer?

The answer is 52.

Problem 24

Given that the prime factorization of 123123123 is $3^2 \cdot x \cdot y$, with $x < y$, find y.

Problem 24

Given that the prime factorization of 123123123 is $3^2 \cdot x \cdot y$, with $x < y$, find y.

The answer is 333667.

Problem 25

Two standard six-sided dice are rolled. If at least one of the dice has a 2 on top, what is the probability that the dice sum to 9?

Problem 25

Two standard six-sided dice are rolled. If at least one of the dice has a 2 on top, what is the probability that the dice sum to 9?

The answer is 0.

Problem 26

100 Berkeley students are taking a math class. If 77 of them are math majors, 43 of them are statistics majors and 2 of them are computer science majors, what is the greatest number of students who are only music majors that could be in the class if no student is a triple major?

Problem 26

100 Berkeley students are taking a math class. If 77 of them are math majors, 43 of them are statistics majors and 2 of them are computer science majors, what is the greatest number of students who are only music majors that could be in the class if no student is a triple major?

The answer is 23.

Problem 27

Evaluate $47 \cdot 89$.

Problem 27

Evaluate $47 \cdot 89$.

The answer is 4183.

Problem 28

Two trains are approaching each other at 40 mph and 60 mph respectively. If they are currently 70 miles apart, in how many minutes will they meet?

Problem 28

Two trains are approaching each other at 40 mph and 60 mph respectively. If they are currently 70 miles apart, in how many minutes will they meet?

The answer is 42.